# Tutorial on the Use of Significant Figures

All measurements are approximations--no measuring device can give perfect measurements without experimental uncertainty. By convention, a mass measured to 13.2 g is said to have an absolute uncertainty of 0.1 g and is said to have been measured to the nearest 0.1 g. In other words, we are somewhat uncertain about that last digit--it could be a "2"; then again, it could be a "1" or a "3". A mass of 13.20 g indicates an absolute uncertainty of 0.01 g.

The objectives of this tutorial on significant figures are:

- Explain the concept of signficant figures.
- Define rules for deciding the number of significant figures in a measured quantity.
- Explain the concept of an exact number.
- Define rules for determining the number of significant figures in a number calculated as a result of a mathematical operation.
- Explain rules for rounding numbers.
- Present guidelines for using a calculator.
- Provide some exercises to test your skill at significant figures.

**What is a "significant figure"?**

The number of significant figures in a result is simply the number of figures that are known with some degree of reliability. The number 13.2 is said to have 3 significant figures. The number 13.20 is said to have 4 significant figures.

**Rules for deciding the number of significant figures in a measured quantity:**

(1) All nonzero digits are significant:
1.234 g has 4 significant figures,
1.2 g has 2 significant figures.

(2) Zeroes between nonzero digits are significant:
1002 kg has 4 significant figures,
3.07 ml has 3 significant figures.

(3) Zeroes to the left of the first nonzero digits are not significant; such zeroes merely indicate the position of the decimal point:
0.001 $^{o}$C has only 1 significant figure,
0.012 g has 2 significant figures.

(4) Zeroes to the right of a decimal point in a number are significant:
0.023 ml has 2 significant figures,
0.200 g has 3 significant figures.

(5) When a number ends in zeroes that are not to the right of a decimal point, the zeroes are not necessarily significant:
190 miles may be 2 or 3 significant figures,
50,600 calories may be 3, 4, or 5 significant figures.

The potential ambiguity in the last rule can be avoided by the use of standard exponential, or "scientific," notation. For example, depending on whether 3, 4, or 5 significant figures is correct, we could write 50,6000 calories as:
$5.06 \times 10^4$ calories (3 significant figures)
$5.060 \times 10^4$ calories (4 significant figures), or
$5.0600 \times 10^4$ calories (5 significant figures).

**What is a "exact number"?**

Some numbers are exact because they are known with complete certainty.

Most exact numbers are integers: exactly 12 inches are in a foot, there might be exactly 23 students in a class. Exact numbers are often found as conversion factors or as counts of objects.

Exact numbers can be considered to have an infinite number of significant figures. Thus, number of apparent significant figures in any exact number can be ignored as a limiting factor in determining the number of significant figures in the result of a calculation.

**Rules for mathematical operations**

In carrying out calculations, the general rule is that the accuracy of a calculated result is limited by the least accurate measurement involved in the calculation.

(1) In addition and subtraction, the result is rounded off to the last common digit occurring furthest to the right in all components. For example,
100 (assume 3 significant figures) + 23.643 (5 significant figures) = 123.643,
which should be rounded to 124 (3 significant figures).

(2) In multiplication and division, the result should be rounded off so as to have the same number of significant figures as in the component with the least number of significant figures. For example,
3.0 (2 significant figures ) × 12.60 (4 significant figures) = 37.8000
which should be rounded off to 38 (2 significant figures).

**Rules for rounding off numbers**

(1) If the digit to be dropped is greater than 5, the last retained digit is increased by one. For example,
12.6 is rounded to 13.

(2) If the digit to be dropped is less than 5, the last remaining digit is left as it is. For example,
12.4 is rounded to 12.

(3) If the digit to be dropped is 5, and if any digit following it is not zero, the last remaining digit is increased by one. For example,
12.51 is rounded to 13.

(4) If the digit to be dropped is 5 and is followed only by zeroes, the last remaining digit is increased by one if it is odd, but left as it is if even. For example,
11.5 is rounded to 12,
12.5 is rounded to 12.

This rule means that if the digit to be dropped is 5 followed only by zeroes, the result is always rounded to the even digit. The rationale is to avoid bias in rounding: half of the time we round up, half the time we round down.

**General guidelines for using calculators**

When using a calculator, if you work the entirety of a long calculation without writing down any intermediate results, you may not be able to tell if a error is made and, even if you realize that one has occurred, you may not be able to tell where the error is.

In a long calculation involving mixed operations, carry as many digits as possible through the entire set of calculations and then round the final result appropriately. For example,

$(5.00 / 1.235) + 3.000 + (6.35 / 4.0) = 4.04858... + 3.000 + 1.5875 = 8.630829...$

The first division should result in 3 significant figures; the last division should result in 2 significant figures; the three numbers added together should result in a number that is rounded off to the last common significant digit occurring furthest to the right (which in this case means the final result should be rounded with 1 digit after the decimal). The correct rounded final result should be 8.6. This final result has been limited by the accuracy in the last division.

Warning: carrying all digits through to the final result before rounding is critical for many mathematical operations in statistics. Rounding intermediate results when calculating sums of squares can seriously compromise the accuracy of the result.

**Sample problems on significant figures**

Instructions: Work the following problems using the correct number of significant figures.

1. $37.76 + 3.907 + 226.4 = ...$
2. $319.15 - 32.614 = ...$
3. $104.630 + 27.08362 + 0.61 = ...$
4. $125 - 0.23 + 4.109 = ...$
5. $2.02 \times 2.5 = ...$
6. $600.0 / 5.2302 = ...$
7. $0.0032 \times 273 = ...$
8. $(5.5)^3 = ...$
9. $0.556 \times (40 - 32.5) = ...$
10. $45 \times 3.00 = ...$
11. What is the average of 0.1707, 0.1713, 0.1720, 0.1704, and 0.1715? ...

**Answer key to sample problems on significant figures**

1. 37.76 + 3.907 + 226.4 = 268.1
2. 319.15 - 32.614 = 286.54
3. 104.630 + 27.08362 + 0.61 = 132.32
4. 125 - 0.23 + 4.109 = 129
5. 2.02 × 2.5 = 5.0
6. 600.0 / 5.2302 = 114.7
7. 0.0032 × 273 = 0.87
8. $(5.5)^3 = 1.7 \times 10^2$
9. 0.556 × (40 - 32.5) = 4
10. $45 \times 3.00 = 1.4 \times 10^2$

11. What is the average of 0.1707, 0.1713, 0.1720, 0.1704, and 0.1715? Answer = 0.1712

12. Calculate the sum of the squares of the deviations from the mean for the five numbers given in Question 11 above, in two different ways: (a) carrying all digits through all the calculations;
(b) rounding all intermediate results to 2 significant figures after subtracting the mean from each (on your way to calculating the sum of the squares about the mean). Compare the two results.
Answer: They are drastically different!